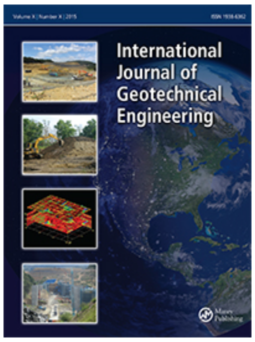

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State of the art: Three Dimensional (3D) Slope-Stability Analysis

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Stability of slope is a major problem in the area of geotechnical engineering. The analysis and design of failing slopes and highway embankments requires an in-depth understanding of the failure mechanism in order to choose the right slope-stability analysis method. Two-dimensional (2D) slope-stability methods are the most commonly used methods among the engineers due to their simplicity. However, the assumptions used in these 2D methods are very simple which reduces the 3D problem to a 2D problem and hence the accuracy of the analysis results vary between the different analysis methods. The importance of 3D analysis of slope stability is greatly increased where the geometry is complex which makes it difficult to select a typical section for 2D analysis. Since 1990s, a lot many number of three-dimensional slope stability-analysis methods were developed based on limit equilibrium and finite element concept. Many of them are valid only under certain conditions. This paper basically focuses on the literature review of 3D slope stability by various researchers.

Keywords: Geotechnical engineering, Slope stability, Failure mechanism, Highway embankments, Finite element method, Limit equilibrium method

1. Introduction

Three-dimensional slope-stability problems are rarely used in practice as they are more complicated than the two-dimensional methods. However, a two-dimensional analysis can be regarded as conservative where 3D failure is expected and it is often preferred in design (Cornforth, 2005). The three-dimensional slope-stability analysis marks its importance where the nature of the slope is highly complex and it is difficult to select a two-dimensional plane strain analysis. The importance of three-dimensional slope-stability analysis can be found where the slope geometry and the slip surface differs significantly in the lateral direction, the material properties are highly non-homogenous and anisotropic, the slope is locally surcharched and to back calculate the shear strength of a failed slope. It was Dr Fredlund in 1970 who illustrated the benefits associated with performing 3D slope stability. In the recent years, many 3D slope-stability methods were researched ranging from method of columns based on variational calculus to the use of dynamic programming. The increase in the importance of three-dimensional slope stability is due to the fact that most of the slope failures are three-dimensional in nature having a dish-shaped failure surface. Like the 2D methods, the 3D methods also require some assumptions to achieve a statically determinate definition of the problem. Some 3D methods do it by decreasing the number of unknowns, while some others achieve it by increasing the number of equations or both, such that the two numbers tally with each other. A literature review of the various researches done in the field of 3D slope stability has been discussed below based on two approaches: Limit Equilibrium Approach and Finite Element Approach.

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2. Literature review

2.1. Based on limit equilibrium approach

The 3D slope stability have been used since 1969. Almost all the 3D limit equilibrium methods (LEMs) were extended from 2D slice methods. The first 3D slope-stability method to calculate the Factor of Safety (FOS) was given by Anagnosti (1969). This method was an extension of earlier Morgenstern and Price's (1965) method. A similar procedure of determining the 3D FOS was done by Sun *et al.* (2011). Hungr (1987), Hungr *et al.* (1989), Ugai (1988), Huang *et al.* (2002) and Cheng and Yip (2007) also extended the 2D LEMs to develop the 3D methods for determining the FOS. The various 2D LEMs include Fellenius method (1936), Simplified Janbu method (1954), Bishop's method (1955), Generalized Janbu method (1957) and Morgenstern and Price's method (1965). The assumptions of each of these 3D methods followed the corresponding assumptions of its 2D origin, but the slip surface was assumed different for different slopes. Some researchers assumed it to be a rotational surface of circular cross-section and some others assumed cylindrical cross-section. The FOS obtained by the 3D methods was found to be higher than the 2D methods. Chen *et al.* (2003) presented a simplified 3D slope-stability analysis based on limit equilibrium theory which is basically an extension of Spencer's (1967) method. A parallel inter-column force inclination was assumed similar

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to Spencer's 2D method. This assumptions satisfies both the force equilibrium and the moment equilibrium requirement about the main axis of rotation. Jiang and Yamagami (2004) also extended the Spencer's (1967) method and assumed the direction of shear to be perpendicular to the longitudinal extent of the slope. They established two different equations for FOS one with respect to horizontal force and the other with respect to overall moment equilibriums. The FOS was then determined by simultaneous solving of these equations with different values for inclination of inter-column forces. The intersection point of two resultant plots, achieved from two equations of FOS, resulted overall FOS.

2.1.1. Slope stability by LEM for cohesive soil

Many researchers have done many researches in the field of 3D slope stability for cohesive soil. Baligh and Azzouz were the first to present a 3D method for cohesive soil based on circular arc method in 1975 where the slip surface was assumed to be a combination of cylindrical centre part with conical ends. Hovland (1977) also performed a similar kind of work on 3D slope stability of cohesive soil. His method was basically an extension of ordinary method of slices, but the method ignored all the inter-column forces on the sides of the columns and the pore water pressure. The conclusion of his work indicated that for cohesive soils, the 3D FOS is always higher than the 2D FOS. Azzouz and Baligh again in 1978 made an attempt to expand their previous work of 1975. The assumptions related to shear resistance force did not change; rather, two new assumptions were introduced for the distribution of other forces. The first assumption followed the method of slices (Fellenius, 1936) and to calculate the normal stresses based on moment equilibrium of each slice and the second one assumed that the vertical effective stress is the major principal stress and the horizontal stress is the minor principal stress. Based on the analysis of four embankments done by Azzouz and Baligh, they concluded that their new assumptions provide more reasonable results than the previous assumptions of slope-stability method. Chen and Chameau (1983a, 1983b) presented a 3D method to analyse symmetrical homogenous cohesive and frictional slope. Chen and Chamaeu considered both the force and moment equilibrium and different pore water pressure conditions in the analysis. They finally found the 3D FOS to be higher than the 2D FOS in the presence of pore water pressure. A similar work on cohesive soil was done by Gens *et al.* in 1988 where the slip surface was assumed to be similar to the work done by Azzouz and Baligh (1983). The assumed slip surface was a combination of cylindrical centre part followed by curve ends to calculate the FOS. The results of Gens *et al.* showed that the ratio of the 3D FOS to the 2D FOS is more than unity and varies from 1.03 to 1.30.

2.1.2. Slope stability by LEM having external loads

Many other research methodologies include the application of various loading conditions to know the effect of loading conditions on the stability of soils. Azzouz and Baligh (1983) extended the 3D method of their previous work of 1975 to consider the effect of applied loads on the stability of slopes. The geometry of the slope remained simple and the slip surface was assumed to be a combination of central cylindrical and ellipsoids at the ends. A comparative study was made between 2D and 3D slope-stability problems due to the distribution of local loads for finding the FOS. All numerical procedures were kept similar to their previous method of 1975. From the several practical cases conducted by Azzouz and Baligh, they finally come to the conclusion that the effect of 3D analysis could increase the capacity of critical load of 2D analysis between 5 and 10 times. Again Dennhardt and Forster (1985) proposed a 3D model to find the FOS of symmetrical slopes with ellipsoidal slip surface by considering a symmetrical external load on top of the slope. Dennhardt and Forster assumed a distribution of normal stress throughout the slip surface to overcome the indeterminacy of the problem. The calculated 3D FOS by this method was found to be higher than corresponding 2D factor.

2.1.3. Slope stability for seismic condition

Very few 3D methods have been established based on limit analysis method to determine the seismic stability of slopes. Some of the recent studies on 3D seismic slope stability have been summarised. Ganjian *et al.* (2010) proposed a 3D method based on upper bound theorem of limit analysis to determine the seismic stability of slopes under local loading. Using the proposed 3D rotational collapse mechanism and applying the energy dissipation method, seismic stability factors for non-associated slopes were determined, and then the effects of dilatancy angle on the stability of locally loaded slopes were investigated. On comparing the results with other analytical and numerical methods, they finally came to a conclusion that the dilatancy angle is more important in 3D seismic analysis of locally loaded slopes. Nadukuru *et al.* (2011) developed a 3D slope-stability analysis with quasi-static distributed force. The charts developed for calculating the FOS was found to be very advantageous as it does not need any iterative procedure. The analysis is found to be applicable in cases where the width of the mechanism is found to be limited or when the mechanism is confined by local geology. They also developed an analysis for calculations of critical acceleration coefficient and displacements due to seismic excitation. However, the analysis was found to have a limitation that it is applicable to slopes of inclination not smaller than 45°. Michalowski and Martel (2011) carried out a 3D slope-stability analysis limited to steep slopes based on the kinematic theorem of limit analysis. A rotational failure mechanism is used with the failure surface in the shape of a curvilinear cone sector passing through the slope toe, typical of steep slopes. Based on quasi-static approach, stability charts were developed to calculate the safety factor and the charts were found to have high practical importance as it does not require any iterative procedure for estimating the FOS. Nadukuru and Michalowski continued to work on the kinematic theorem of limit analysis and in 2013 developed a method to calculate the yield acceleration of slopes that fail in a 3D manner, with an assumed width of the mechanism. An analysis was then carried out to arrive at the displacements of slopes subjected to ground shaking. The outcome of the analysis was found to be very convenient to use in practical applications. Tiwari *et al.* (2015) uses Specfem 3D Slope, an open source spectral element method (SEM)-based program to evaluate the stability of large-scale landslides. SEM technique is highly beneficial as it drastically reduces the huge computational burden if it proceeds for p-refinement techniques instead of h-refinement unlike FEM. Hence, the SEM technique seemed to be very accurate and powerful in elasto-plastic modelling which deserves unique computational scheme in large-scale landslide modelling.

2.1.4. Slope stability for unsaturated soil

Slope-stability analysis of unsaturated soil requires to simultaneously compute deformation and groundwater flow with time-dependent boundary conditions. Very little work has been found to be done on the 3D slope stability of unsaturated soil. Li *et al.* (2006) described the implementation of strength reduction technique method for slope-stability analysis using Finite Element Method (FEM). Strength reduction FEM can take into account non-uniform distribution of metric suction and therefore has distinctive advantages in dealing with 3-D stability of unsaturated soil slope compared to conventional methods. Strength reduction FEM has been found to be a reliable numerical approach compared to the stability-analysis of unsaturated slope. Yong *et al.* (2010) computed the pore water pressure fields of unsaturated seepage to ascertain the stress and strain distributions of 3D slope based on the constitutive model of unsaturated soils. The contribution of matric suction to shear strength is considered and three-dimensional stability analysis methods for unsaturated soil slopes were performed to evaluate the FOS. Based on the results of three-dimensional unsaturated seepage analysis, the variation rules of pore water pressure of six observation points are in accordance with the change in reservoir level, and compared to their changes, the pore water pressure shows some hysteresis at different degrees. Zhang *et al.* (2015) carried out a comparative study between 2D and 3D slope-stability analyses for unsaturated slopes and came to a conclusion that for simple slopes with low slope angle, $\Delta Fs/Fs_{2D}$ monotonically increases with an increase in the value of c' and ϕ', whereas the value does not increase for a simple steep slope. The difference of FOS between 2D and 3D analysis for a simple steep slope is found to be larger for a simple slope having lower slope angle. They also found that the difference between 2D and 3D stability analysis was most pronounced for concave geometrics.

2.1.5. Slope stability by computer-aided programs

Many researchers worked on various computer aided and optimisation programs to study the stability of soils. Xie *et al.* (2006) and Tiwari and Douglas (2012) used the GIS grid-based 3D models to study the slope-stability analysis. Shen and Karakus (2013) used the FLAC-3D program to analyse the 3D slope stability. A non-linear shear strength reduction (SSR) technique was proposed that can use the Hoek–Brown (HB) criterion to represent the non-linear behaviour of a rock mass in the FLAC-3D program. The result of the proposed technique found to be very satisfactory. Rashid *et al*. (2015) presented particle swarm optimisation (PSO) in three-dimensional (3D) slope-stability analysis to determine the shape and direction of failure as the critical slip surface and a factor of safety (FOS) was developed based on limit equilibrium method. A coding system was developed in Matlab to work out the 3D form of the failure surface and calculate its FOS. A 3D slope model beneath the vertical load was finally made and tested within the laboratory. The results obtained from PSO were re-analysed and compared with the code results and it was found that the given codes were highly effective in determining the 3D failure surface of the soil slopes.

Stark and Eid (1998) found that the commercially offered 3D slope-stability software doesn't take into account the shear resistance on the two sides of the sliding mass. As a result, the 3D factor of safety may be underestimated whereas the back-calculated shear strength may be overestimated. Arellano and Stark (2000) offered a new technique for incorporating the shear resistance on the two sides of the sliding mass in existing 3D software. Huang and Tsai (2000) developed a new 3D slope-stability method which is based on 2D moment equilibrium method. They found that the new method is very advantageous as it calculates the factor of safety as well as the possible direction of sliding for semispherical and composite failure surfaces. Hence, the errors generated from assuming a plane of symmetry is removed. Again Farzaneh *et al.* in 2008 based on the upper bound theorem of limit analysis presented a new three-dimensional slope-stability analysis for convex slopes. This method has the advantage of calculating both the 3D factor of safety and the bearing capacity of foundations adjacent to such slopes. On comparing the results of bearing capacity of foundations, they came to a conclusion that the one located near the straight slopes has more capacity than the one located near convex slopes. Zheng (2009) presented a rigorous 3D method that considered the whole failure body rather than discretising it into columns. The sliding surface was assumed to possess a general shape with an arbitrary direction of shear. Zheng considered six equilibrium conditions for the sliding mass along with a vector of integration equation. The unknown values of these equations include FOS and total normal stress on the sliding surface that was defined by a distribution form including five unknowns. Then, the distribution function was substituted into the mentioned six equations and provided a system of non-linear equations. The FOS and the distribution vector was found by solving the system of non-linear equations. Michalowski (2010) approaches kinematically to calculate the 3D factor of safety. He prepared stability charts using 3D failure mechanism for finding the factor of safety. These stability charts are very helpful in calculating the factor of safety as it does not require any iteration. Michalowski continued his research work in the field of kinematic approach of limit analysis and in 2013, Nadukuru and Michalowski described a three-dimensional slope-stability analysis applicable to slopes whose geometry of failure patterns was physically constrained**.** Gao *et al.* in 2013 extended a kinematically based 3D method of slope stability. In addition to toe failure, the extended method incorporates face failure and base failure in both purely cohesive and frictional soils. An analytical approach is derived afterwards to obtain the upper bounds on slope stability and the corresponding type of the critical failure mechanism. The results are then compared with a finite element analysis method and on comparing the results they found that the 3D rotational failure mechanisms give the best estimate on the upper bound. Zhou and Pond (2013) presented a rigorous approach for slot-cut stability analysis that is applicable for slopes even with complex geometry, stratigraphy and surcharge loading conditions. They considered the 3D effect by incorporating side-panel shear resistance in the force limit-equilibrium equations. The result found to be very successful and the application of slot-cut construction can be readily applied for removal and repair of a landslide below an existing residence.

2.2. Based on finite element approach

Finite Element Analysis (FEA) uses the FEM to analyse a material or object and find how applied stresses will affect the material or design. FEA is basically used in Mechanical and Civil Engineering fields for analysing and solving complex geometrical problems. The methods most commonly used at present for slope-stability analysis are the rigid-body LEM and the FEM (Bishop, 1955; Duncan, 1996; Griffiths and Lane, 1999 and Chen *et al.*, 2005). The former yields a safety factor determined by analysing the limit equilibrium status of a block. The method is characterised by simple calculations. However, LEM cannot take non-linear structural deformation into account and the method assumes that sliding surfaces reaches an ultimate state of failure simultaneously, which does not reflect the actual stress status of slip surfaces (Lenchman and Griffiths, 2000). FEM can be used to determine the stress field and displacement field of the slope but cannot yield a specific value for the slope-stability safety factor (Liu *et al.*, 2008). Although many researchers have obtained slope stability safety factors using the strength reduction method together with FEA (Jiang and Magnan, 1997; Dawson *et al.*, 1999 and Zhao *et al.*, 2002). Jeramic (2000) presented a new approach for modelling of three-dimensional slope-stability problems. A p-version of the FEM together with large deformation hyper elastic–plastic formulation is utilised to model localised, continuous deformation that has been observed in failure mechanisms of slopes. In particular, it is shown how the new method can be used with a rather small number of finite elements to model sharp deformation gradients resulting from shear localisation during slope failures. Tan and Sharma (2008) developed a new method for both homogenous and non-homogenous slopes based on limit equilibrium method and used FEM to validate the new procedure. The procedure was found to be in satisfaction with the FEM. However, some differences were noticed when non-associated flow rules were adopted in FEA. Li and Shao in 2011 presented a three-dimensional finite element limit equilibrium method (3DFLEM) based on the concept of strength reduction and the unique sliding direction. They also clarified the physical meaning of factor of safety as well as the relationship of FOS and the unique sliding direction. They compared the results of stability analysis obtained from the proposed approach, 3D rigid limit equilibrium method (3DLEM) and 3D shear strength reduction method (3DSSRM) and found that the FOS and critical sliding surface are generally in good agreement and the element size of slope and sliding surface has certain effect on FOS by causing maximum difference of 2%. Nian *et al.* (2012) analysed a 3D slope-stability method using finite element strength reduction method. They found that the concave-shaped vertical slope with a 90° corner angle is markedly higher than that of a convex-shaped vertical slope with a 90° corner angle. Moreover, they also found that a concave-shaped vertical slope with a 90° corner angle can be replaced by a straight vertical slope for computation of the FOS. Liu *et al.* (2013) used the multi-grid method to establish two grids, a structural grid for finite element computation and a sliding surface grid for calculating a sliding surface's stability safety factor. This combination of grids makes it easy to determine the stability safety factor of any sliding surface or sliding block, and it also considers the influences of non-linear deformation and elastic–plastic stress adjustment on the stability safety factor. Zhang *et al.* (2013) analysed the effects of complex geometrics on 3D slope stability using an elastoplastic finite difference method (FDM) with a strength reduction technique. The results obtained from the analyses were useful for landslide hazard preparedness or safe and economical design of infrastructures. Kelesoglu (2015) in his paper investigated the effect of each contributing factor such as the curvature of the slope, the contribution of the piles and the local loading of the slope by using SRM (Strength Reduction Method) and FDM. He found that the stability of concave slopes is higher than those straight slopes. The FOS values are increased up to 15 to 25% for slopes that have sharp concave curvatures and 5 to 10% for smooth concave curvatures compared to a straight slope. He also found that when there is local loading on top of the slope, due to the mobilised shear strains under the surcharge, the pile row must move uphill towards the load to ensure the local and global stability of the slope. If the surcharge is next to the crest ($b = 0$ m), then the effective pile location is adjacent to the slope crest. If the surcharge moves away from the crest ($b > 0$ m) the effective pile location is located between the no surcharge case and the crest of the slope. In this case, if the pile row is located within these boundaries then the FOS values differs only marginally.

3. Conclusion

Slope-stability analysis is an extremely important as the result of slope failure can often be catastrophic, involving considerable loss of life and property. Hence, slope instability is widely recognised as an ever-present danger and it is a continual source of concern for geotechnical engineers and engineering geologists throughout the world. Different methods of slope-stability analysis have been developed in the past and each of them has advantages and limitations over the others. In the present study, literature review is carried out on the three-dimensional slope-stability analysis based on two approaches – limit equilibrium approach and finite element approach. The literature study of the limit equilibrium approach reveals that most of the 3D slope stability LEMs are derived from 2D LEMs. The assumptions of the 3D slope-stability methods were kept similar to the 2D methods but the 3D FOS is found to be higher than the 2D FOS. Many literatures were studied where the 3D FOS was determined for cohesive soil and surcharge loading conditions. It was found that for both the cases the 3D FOS was found to be higher than the 2D FOS. Very little work has been done on 3D seismic slope stability and 3D slope stability for unsaturated soil. In majority of the methods, charts have been prepared to calculate the FOS under seismic conditions which is found to be very helpful as it eliminates the calculation of iterative method. The literature study of the FEM approach reveals that shear strength reduction technique is mostly employed to analyse the stability of slopes. Studies were also found to be done based on 3DFLEM, multi grid method, elastoplastic finite difference method, etc. It is found that less attention is given towards the delayed and/or time-dependent behaviour of slopes; hence, further research work needs to be done to know the time-dependent behaviour of slopes. More emphasis have to be placed on 3D slope stability for seismic and unsaturated soil conditions.

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