

Prediction of slope stability using multiple linear regression (MLR) and artificial neural network (ANN)

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Abstract The stability problem of natural slopes, filled slopes, and cut slopes are commonly encountered in Civil Engineering Projects. Predicting the slope stability is an everyday task for geotechnical engineers. In this paper, a study has been done to predict the factor of safety (FOS) of the slopes using multiple linear regression (MLR) and artificial neural network (ANN). A total of 200 cases with different geometric and shear strength parameters were analyzed by using the well-known slope stability methods like Fellenius method, Bishop's method, Janbu method, and Morgenstern and Price method. The FOS values obtained by these slope stability methods were used to develop the prediction models using MLR and ANN. Further, a few case studies have been done along the Jorabat-Shillong Expressway (NH-40) in India, using the finite element method (FEM). The output values of FEM were compared with the developed prediction models to find the best prediction model and the results were discussed.

Keywords Slope stability · Multiple regression analysis · Artificial neural network · Shear strength · Finite element method

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Introduction

The stabilization of slopes are causing a major problem to the geotechnical engineers. Prediction of the slope stability is the main challenge for the geotechnical engineers because the stability of the slopes generally exists as the combined effects of geology, hydrology, and soil parameters. Because of its practical importance, slope stability analysis has drawn the attention of many investigators. Many investigators have studied about the prediction analysis of slope stability by using various prediction methods. Sakellariou and Ferentinou (2005) studied on the idea of prediction analysis and used artificial neural network (ANN) to develop a relationship between the various slope parameters. The validation of the ANN model was done by comparing the results with Hoek and Bray (1981) model and results were found to be very satisfactory. Kayesa (2006) used the Geomos slope monitoring system (GSMS) to study the slope stability prediction of Letlhakane mine. The GSMS is basically an automatic and continuous slope monitoring system which runs continuously for 24 h. The system consists of three parts, viz, collection of data, transmission of data, and processing and analysis of data. The GSMS resulted into avoiding potentially fatal injury, damage to mining equipment, and loss of mining production. Ahangar-Asr et al. (2010) developed a prediction model based on evolutionary polynomial regression (EPR) technique to predict the FOS. The EPR models are developed from the results of field data for conditions not used in the model building process, and the results were found to be very effective in modeling the behavior of slopes. Mohamed et al. (2012a) used fuzzy logic system for the prediction of slope stability. They used Geo studio for analyzing and computing the FOS of various slopes. The results were compared with the predicted results obtained from fuzzy logic, and the results were found to be very close to the target data. Mohamed

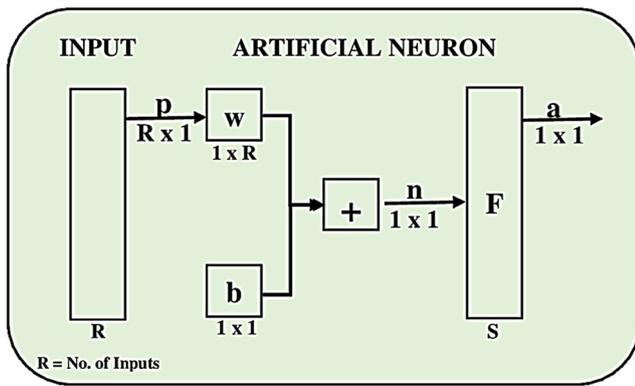


Fig. 1 Artificial neuron model

et al. (2012b) again compared the results of safety factors with the predicted values obtained from adaptive neuro fuzzy inference system (ANFIS) and Multiple linear regression (MLR) and the results showed that ANFIS could predict the safety factors with higher accuracy compared with MLR. Erzin and Cetin (2012) used ANN and MLR for finding the critical value of FOS for a typical artificial slope which is subjected to earthquake forces. The predicted results from both the methods were compared with the calculated results and found that the results obtained from ANN are having a higher degree of precision when compared to MLR. Chae et al. (2015) used the saturation depth ratio to develop a modified equation for rainfall induced slopes. On comparing the results with the landslide inventory graph and previous steady-state hydrological model they found that the proposed approach showed a very satisfactory result in classifying landslide susceptibility and showed better performance than the steady-state approach. Firmansyah et al. (2016) studied with different soil types to predict the run-out distance of a rotational slope using the concept of center of mass approach. They found that the soil unit weight can influence to a great extent the depth of sliding zone and the volume of unstable material.

In this paper, prediction models were developed using MLR and ANN to predict the slope stability and

few case studies have been done to validate the prediction models.

Multiple linear regression (MLR)

In statistics, regression analysis is a statistical tool for predicting the nature of relationship among different variables. The general purpose of MLR is to learn more about the relationship between several independent or predictor variables and a dependent or criterion variable (Yilmaz and Yuksek 2008). This technique is widely used in predicting slope failures and landslides (Pradhan 2010a, b). Simple linear regression is the prediction of a single criterion value which is obtained from one predictor variable whereas in multiple regression, the criterion is predicted by two or more variables. So a multiple regression deals with the examination of correlations between multiple independent variables and dependent variable. The general equation for multiple regression is

$$Y = a + b_1 \times x_1 + b_2 \times x_2 + b_3 \times x_3 + \dots + b_n \times x_n + \epsilon$$

where

- Y dependent variable
- $x_1, x_2, x_3, \dots, x_n$ independent variable
- $b_1, b_2, b_3, \dots, b_n$ regression coefficient
- a constant
- ϵ error.

In this equation, the regression coefficients represent the independent contributions of each independent variable to the prediction of the dependent variable. The regression line expresses the best prediction of the dependent variable (Y), given the independent variables (X). However, the nature is rarely perfectly predictable, and hence, there is always a substantial variation of the observed points around the fitted regression line. The deviation of a particular point from the regression line is called the residual value. R square, also known as the coefficient of determination, is used to evaluate model fit which is given by 1 minus the ratio of residual variability. When these residual values are having a small

Fig. 2 Multi-layer feed-forward network

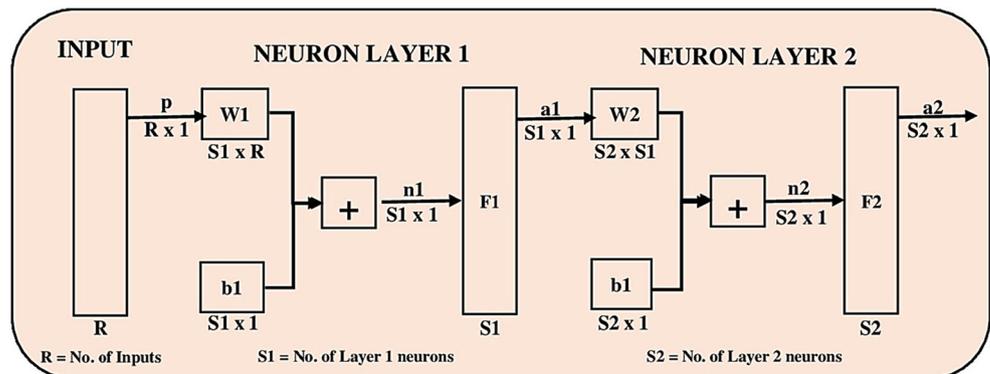
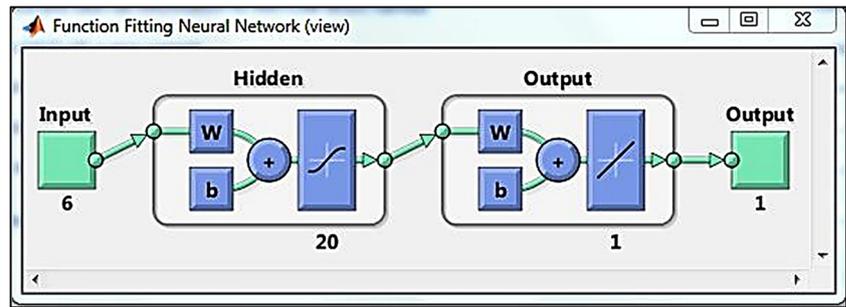


Fig. 3 Neural network showing hidden neurons



variability around the regression line, the predictions obtained from the regression equation are good. In other words, as the value of correlation coefficient R , (square root of R square) approaches unity, the relationship between the variables grows stronger. Smith (1986) suggested the following guide for values of $|R|$ between 0.0 and 1.0:

- $|R| \geq 0.8$ strong correlation exists between two sets of variables,
- $0.2 < |R| < 0.8$ correlation exists between the two sets of variables, and
- $|R| \leq 0.2$ weak correlation exists between the two sets of variables.

Artificial neural network

An artificial neural network (ANN) is a computational model that works similar to the neurons present in the brain. It consists of an interconnected assembly of artificial neurons, which transmits the information through the tendons present in the neurons. ANN acts as a powerful tool for modeling, especially when the relationships between the underlying data is unknown. It can identify and understand the correlated patterns present between the input data sets and corresponding target values. The neuron model and the network architecture enlighten how a network transmutes its input into an output (Gupta et al. 2003). The way a network computes its output must be understood before training methods for the network can be explained. Let us consider a single artificial neuron with R inputs as shown in the Fig. 1.

Here, the input vector p (a column vector, $R \times 1$) is shown by a vertical bar on the left. These inputs go to the row vector w of size $1 \times R$. The net input n given by the sum of bias b and the product $w \times p$ is passed to the transfer function F to obtain the neuron’s output. Depending upon the nature of the problem, the transfer function F can be linear or sigmoidal. The sigmoidal transfer function is commonly used in multiple-layer networks (McClelland and Rumelhart 1986; Demuth and Beale 1995). In the multi-layer network, the outputs of the intermediate layer are the inputs to the following layer. Thus, layer 2 can be analyzed as a single-layer network with $R = S1$ inputs, $S = S2$ neurons,

and weight matrix $w = (S1 \times S2)$. The input to the layer 2 is $p = a1$ and the output is $a = a2$. The layer of a multi-layer network plays a different role. A layer that produces the network output is called an output layer while all other layers in the network are called the hidden layers. The two layer network shown in Fig. 2 has one output layer and one hidden layer.

Multi-layer networks are much powerful compared to single-layer networks as they are capable of using the combination of sigmoidal and/or linear transfer function. The process of optimizing the connection weights is known as training. The most widely used training method for multi-layer neural feed-forward networks is Levenberg-Marquardt back-propagation algorithm (Rumelhart et al. 1986). The stopping criteria are considered to be the most important criteria and are used to stop the training process. They determine whether the model has been trained optimally (Maier and Dandy 2000). Training can be stopped after the presentation of a fixed

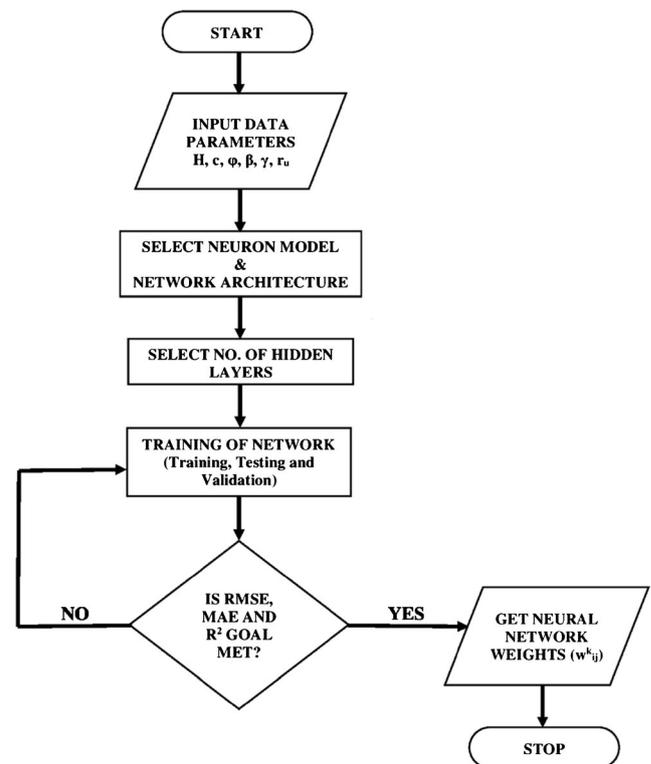


Fig. 4 Flowchart showing determination of neural network weights

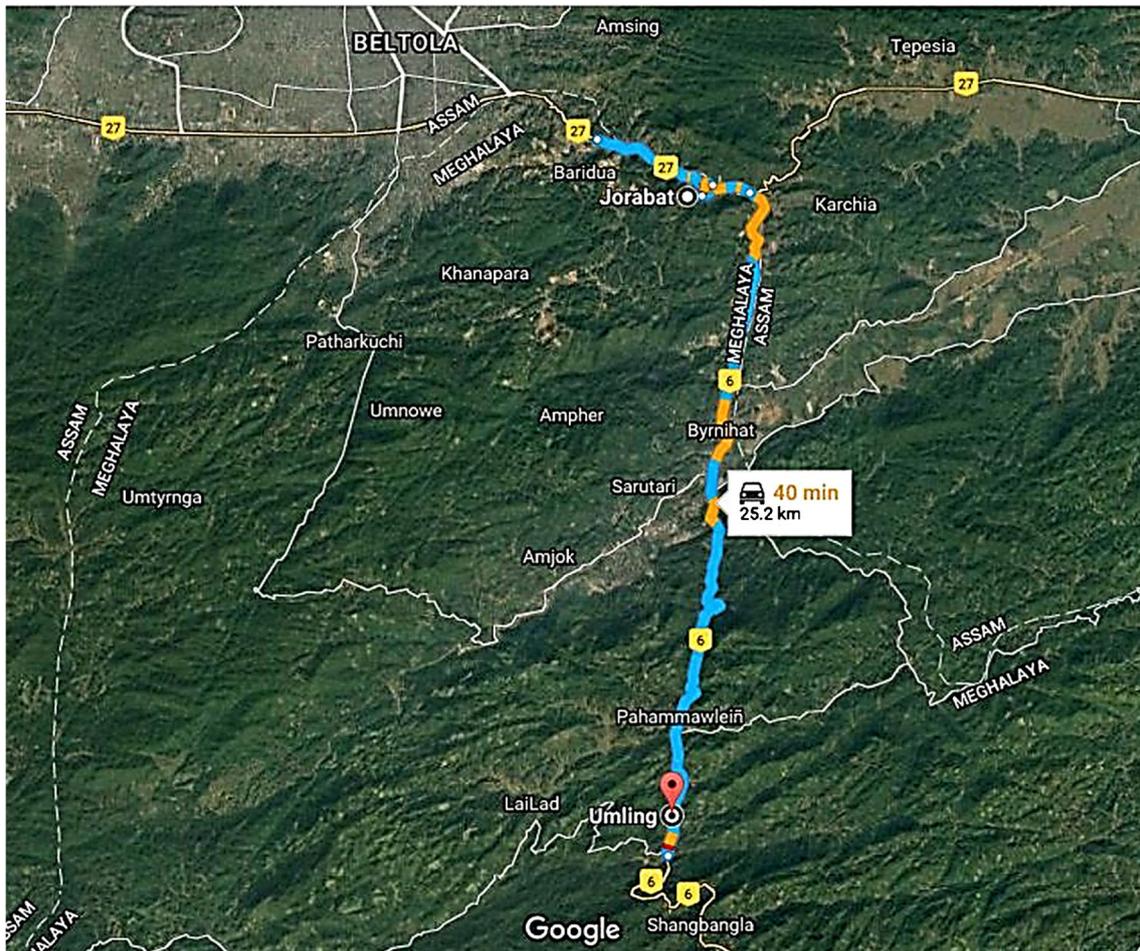


Fig. 5 Satellite-based image map of study area of NH-40

number of training records, when the training error reaches a sufficiently small value, or when no or slight changes in the training error occur. However, the above techniques of

stopping criteria may lead to the model stopping prematurely or over training. Such problems overcome with the use of cross-validation technique (Stone 1974). The cross-

Table 1 Summary of MLR for 200 cases for Bishop’s method

Summary output of MLR for Bishop’s method					
Regression statistics					
	Multiple R	0.920588762			
	R square	0.847483670			
	Adjusted R square	0.842742229			
	Standard error	0.138292486			
	Observations	200			
Stability parameters	Coefficients	Standard error	t stat	P value	
	Intercept	2.525560294	0.153975422	14.96057136	4.19856E-34
	H	- 0.047241648	0.003946599	- 11.97021637	4.67491E-25
	c	0.028068049	0.001490302	18.83379414	1.37321E-45
	φ	0.018619905	0.001767191	10.53644183	8.51635E-21
	β	- 0.017053804	0.001432000	- 11.90908428	7.13418E-25
	γ	- 0.032343712	0.006702797	- 4.825405022	2.83183E-06
	r _u	- 0.967234986	0.050786840	- 19.04499241	3.37569E-46

Table 2 Summary of MLR for 200 cases for Fellenius method

Summary output of MLR for Fellenius method				
Regression statistics				
Multiple <i>R</i>		0.913964700		
<i>R</i> square		0.835331473		
Adjusted <i>R</i> square		0.830212245		
Standard error		0.145457008		
Observations		200		
Stability parameters	Coefficients	Standard error	<i>t</i> stat	<i>P</i> value
Intercept	2.529276984	0.161952431	13.49332626	1.16859E-29
<i>H</i>	− 0.048514688	0.004151061	− 11.68729843	3.29757E-24
<i>c</i>	0.029509272	0.001567511	18.82556569	1.45051E-45
φ	0.017297598	0.001858744	9.306068388	2.97359E-17
β	− 0.013740426	0.001506187	− 9.122654264	9.74584E-17
γ	− 0.035033755	0.007050049	− 4.96929230	1.47707E-06
<i>r_u</i>	− 0.977343262	0.053417955	− 18.29615640	4.99098E-44

validation technique requires the data to be divided into three distinct sets, viz, training, testing and validation of which the training set is the biggest set which is used by neural network to identify the patterns present in the data. The objective of training is to find the set of weights (w_{ij}^k) between the neurons that determine the global minimum of error function based on the following relationships:

$$y_i^k = f(y_i^{k-1}) = f\left(\sum_{j=1}^{n_{k-1}} w_{ij}^k \times y_j^{k-1}\right)$$

The main function of the testing set is to evaluate the generalization ability of a trained network, and the validation set performs the final check of the trained network. As suggested by Beale and Jackson (1990), the number of hidden layers in the network can be determined by trial and error technique.

Several ANN models have been examined by varying the number of hidden layers and the number of neurons present in each hidden layers. This helps to identify the best neural network. The behavior of the network is administered by the values of its weights and thresholds which are governed by the training data sets of the network. Once the training phase of the model is successfully completed, the performance of the trained model should be validated. The validation phase of the model is performed to check the generalization ability of the trained model within the limits set by the training data in a robust fashion, rather than simply memorizing the input-output relationships that are contained in the training data. The best approach to validate the trained model is to test the performance of the same on an independent data set, which has not been used as part of the model building process. If such performance is adequate, the model is deemed to be able

Table 3 Summary of MLR for 200 cases for Janbu method

Summary output of MLR for Janbu method				
Regression statistics				
Multiple <i>R</i>		0.908764542		
<i>R</i> square		0.825852993		
Adjusted <i>R</i> square		0.820439096		
Standard error		0.151752898		
Observations		200		
Stability parameters	Coefficients	Standard error	<i>t</i> stat	<i>P</i> value
Intercept	2.667218573	0.168962300	13.34154761	3.37089E-29
<i>H</i>	− 0.051426388	0.004330733	− 11.87475256	9.0444E-25
<i>c</i>	0.031080022	0.001635358	19.00502788	4.40061E-46
φ	0.015972536	0.001939197	8.236676053	2.63409E-14
β	− 0.012747056	0.001571380	− 8.112011726	5.67692E-14
γ	− 0.038265408	0.007355200	− 5.202497151	4.99963E-07
<i>r_u</i>	− 0.925287223	0.055730072	− 16.60301509	4.90659E-39

Table 4 Summary of MLR for 200 cases for Morgenstern-Price method

Summary output of MLR for Morgenstern-Price method

Regression statistics				
Multiple R		0.852297017		
R square		0.726410205		
Adjusted R square		0.717904823		
Standard error		0.223311614		
Observations		200		
Stability parameters	Coefficients	Standard error	t stat	P value
Intercept	2.876814062	0.248636069	10.05008674	2.2213E-19
H	-0.056570127	0.006372880	-8.87669676	4.72104E-16
c	0.035726233	0.002406507	14.84568020	9.34044E-34
φ	0.012725932	0.002853621	4.459574227	1.3921E-05
β	-0.007832119	0.002312361	-3.387065674	0.000856096
γ	-0.058052947	0.010823527	-5.363588622	2.31885E-07
r_u	-0.981904581	0.082009454	-11.97306571	4.58368E-25

to generalize and is considered to be robust. The coefficient of correlation, R ; the root mean squared error, RMSE; and the mean absolute error, MAE, are the main criteria that are often used to evaluate the prediction performance of ANN models.

Methodology

In this research, 200 artificial slopes with different slope parameters were studied and analyzed using the most popular methods of slope stability, viz, Bishop's method, Fellenius method, Janbu method, and Morgenstern-Price method to calculate the factor of safety (FOS). The FOS values obtained by these methods were used to develop the prediction models using MLR and ANN. In the proposed models for predicting slope stability, several important parameters including height of the slope (H), cohesion (c), angle of internal friction (φ), slope inclination (β), unit weight of soil (γ), and coefficient of pore water pressure (r_u) were used as input parameters whereas the FOS was used as the output parameter.

The MLR model for predicting the FOS was developed using Microsoft Excel 2013.

The ANN model was prepared in MATLAB 2011a. Here, multi-layer feed-forward network having 20 neurons in hidden layer and 1 neuron in output layer is used for developing the prediction model which is shown in Fig. 3.

For the cross-validation technique, the whole data set used for the development of the prediction model was divided into three distinct sets, i.e., training, testing, and validation. Out of 200 slope cases, 80% of the data set was used for training and the remaining was used for validating the model. The network was trained up using Levenberg-Marquardt back propagation till the training error reaches a sufficiently small value, or when no or slight changes in the training error occur. In other

words, training is stopped when the regression coefficient R approaches to unity. An R value of 1 means a close relationship and 0 a random relationship. When the value of R of all the three sets, i.e., training, testing, and validation, approaches close to unity, it is assumed that the prediction equation attains a close relationship and the training process is terminated. The flow chart for determination of neural network weights (w_{ij}^k) is shown in Fig. 4.

Case study

Study area

Shillong, the capital of Meghalaya, is one of the smallest states in India having latitude of $25^\circ 34' 32.00''$ N and longitude of $91^\circ 52' 23.00''$ E. It is the headquarters of East-Khasi Hills district and is situated at an average altitude of 1496 m above mean sea level. The expressway connecting Jorabat, Assam, and Shillong, Meghalaya consists of a number of hill slopes, and hence, slope failures are very common in the Jorabat-Shillong expressway (NH-40). According to a survey, it is found that most of the slopes are either in a damp or in a wet condition which creates a lot of troubles especially during the monsoons. Massive landslides have occurred resulting too many calamities during the last two decades. Hence, there is a need to check these frequently occurring landslides. In this research, some case studies have been done to investigate the extent of vulnerability of these border hills. Fifteen vulnerable slopes were selected along NH-40 from Jorabat ($26^\circ 05' 60.00''$ N; $91^\circ 51' 59.99''$ E) to Umling ($25^\circ 58' 24.09''$ N; $91^\circ 51' 31.18''$ E) having a distance of 25 km (Fig. 5), and geotechnical tests were carried out to determine the various soil parameters. These slopes were analyzed using the finite

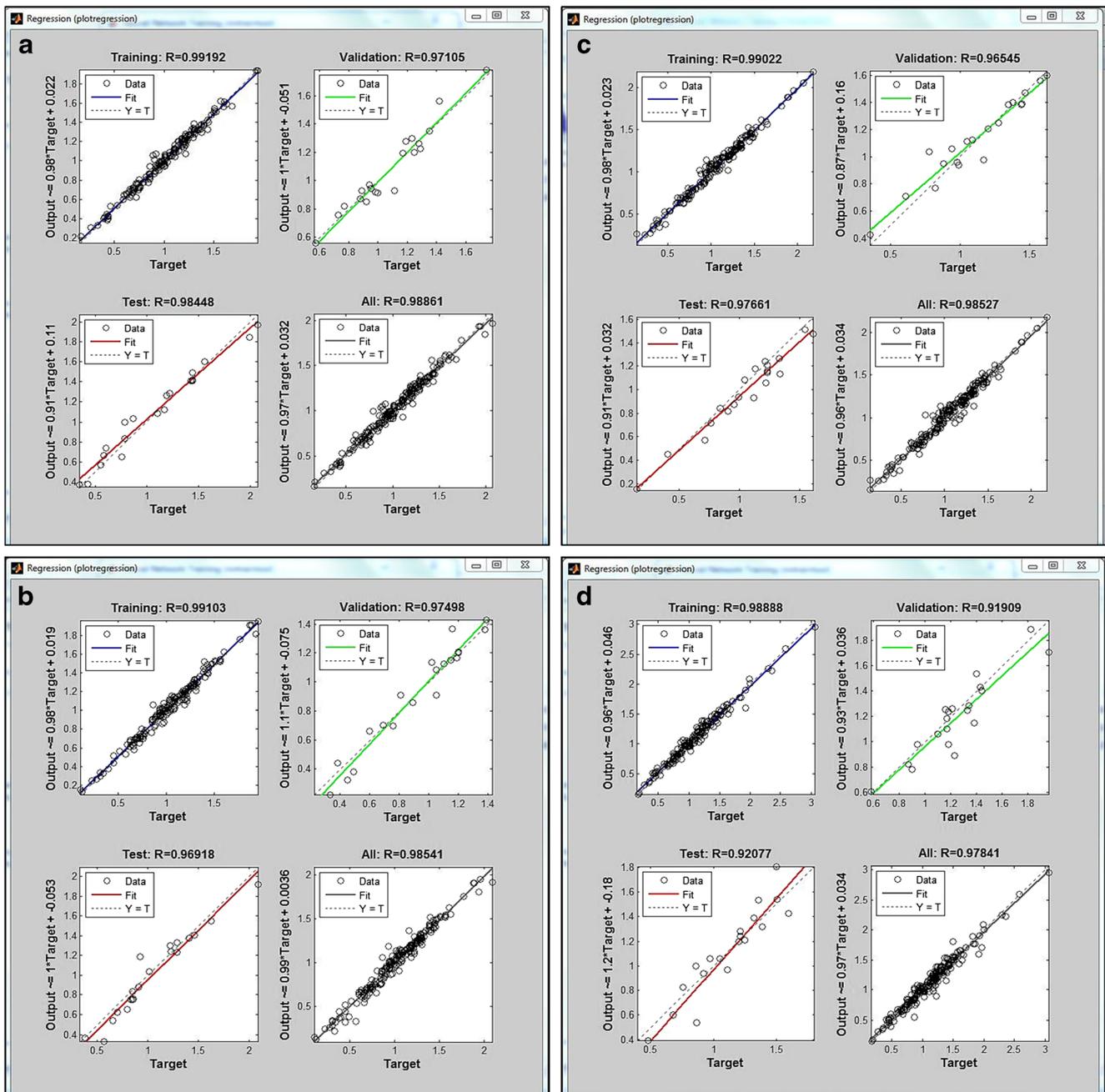


Fig. 6 a Regression plot for Bishop's method. b Regression plot for Fellenius method. c Regression plot for Janbu method. d Regression plot for Morgenstern-Price method

element software PLAXIS to determine the FOS. These analytical results were compared with the values obtained from the prediction models of MLR and ANN, and the results were discussed.

Results and discussion

The FOS values obtained by Bishop's method, Fellenius method, Janbu method, and Morgenstern-Price method were

used to develop MLR and ANN models to obtain the prediction formula for the determination of FOS. The prediction models were validated by comparing the predicted results with the analytical results for 15 vulnerable slope cases from NH-40.

Multiple linear regression

The summary of MLR for 200 artificial slope cases is given in Tables 1, 2, 3, and 4, and it has been found that the Bishop's

Table 5 Random cases for 15 vulnerable slopes

Case study	Predicted results																
	Bishop's method Fellenius method Janbu method M-P method																
	Sl. No.	Chaninage	H	c (kN/m ²)	φ	β	γ (kN/m ³)	r _u	Analytical FOS	MR	ANN	MR	ANN	MR	ANN	MR	ANN
1	08 + 230	38	39.5	30.2	50	17.6	0.04	1.174	0.97	1.11	1.05	1.25	1.08	1.42	1.11	1.45	
2	08 + 620	35	39.0	30.0	50	17.3	0.04	1.190	1.10	1.16	1.18	1.29	1.23	1.22	1.27	1.32	
3	08 + 980	26	38.7	30.5	60	17.8	0.00	1.220	1.38	1.25	1.50	1.34	1.57	1.38	1.70	1.48	
4	09 + 440	25	39.0	31.2	55	17.9	0.15	1.213	1.38	1.28	1.49	1.27	1.57	1.35	1.66	1.52	
5	09 + 530	26	39.0	30.0	50	17.3	0.20	1.388	1.37	1.39	1.46	1.49	1.54	1.62	1.61	1.68	
6	11 + 950	29	37.9	30.0	45	17.3	0.37	1.164	1.12	1.17	1.19	1.03	1.26	1.44	1.28	1.26	
7	12 + 870	33	38.5	29.0	50	17.5	0.20	1.070	1.00	1.03	1.09	1.15	1.14	1.21	1.18	1.15	
8	13 + 780	31	39.2	29.7	55	17.5	0.00	1.171	1.23	1.17	1.34	1.15	1.39	1.37	1.48	1.56	
9	15 + 530	32	39.8	31.3	45	17.8	0.34	1.129	1.07	1.14	1.14	1.20	1.19	1.39	1.20	1.22	
10	15 + 770	30	39.0	30.0	48	17.3	0.03	1.407	1.38	1.41	1.46	1.48	1.52	1.50	1.57	1.49	
11	18 + 460	31	57.2	38.6	38	18.3	0.64	1.657	1.55	1.68	1.61	1.62	1.69	2.10	1.71	1.85	
12	19 + 900	29	5.0	43.5	58	17.4	0.05	0.672	0.54	0.59	0.56	0.65	0.58	0.68	0.48	0.55	
13	19 + 970	31	14.0	44.2	65	17.8	0.07	0.452	0.56	0.43	0.62	0.51	0.65	0.72	0.60	0.72	
14	20 + 140	26	0.0	43.7	60	17.4	0.40	0.236	0.17	0.25	0.18	0.32	0.23	0.42	0.11	0.52	
15	24 + 170	23	57.5	41.3	62	19.8	0.19	1.740	1.97	1.79	2.11	1.85	2.21	1.86	2.37	1.89	

method is having the highest value of *R* square of 0.85 compared to other methods.

Artificial neural network

The regression plot showing the value of *R* for training, testing, and validation is shown in Fig. 6a–d. From the regression plot, it has been found that the value of *R* for Bishop’s method was found to be the highest and equals to 0.99 which is very close to unity. Hence, it can be stated that the prediction results obtained from the Bishop’s method should bear a close relationship between the input variables.

The performance of the predicted models was checked in the validation phase. Here, the validation phase is subdivided into two phases. In the first phase, the efficiency and accuracy of the prediction models were examined by making predictions against case records which were not used during training and testing. The predictions obtained by MLR and ANN are very close to the analytical results. Fifteen vulnerable slope cases were studied and the results of analytical, MLR, and ANN are shown in Table 5. It is evident from Fig. 7 that the correlation of the Bishop’s model for MLR and ANN is found to be over 95% compared to other prediction models. Moreover, it is also evident that the correlation between Bishop’s model and

Fig. 7 Correlation percentage vs type of methods for MLR and ANN

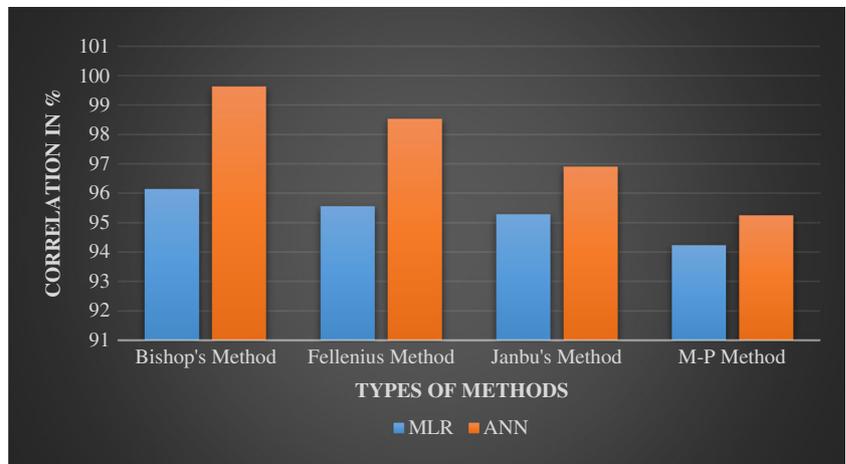
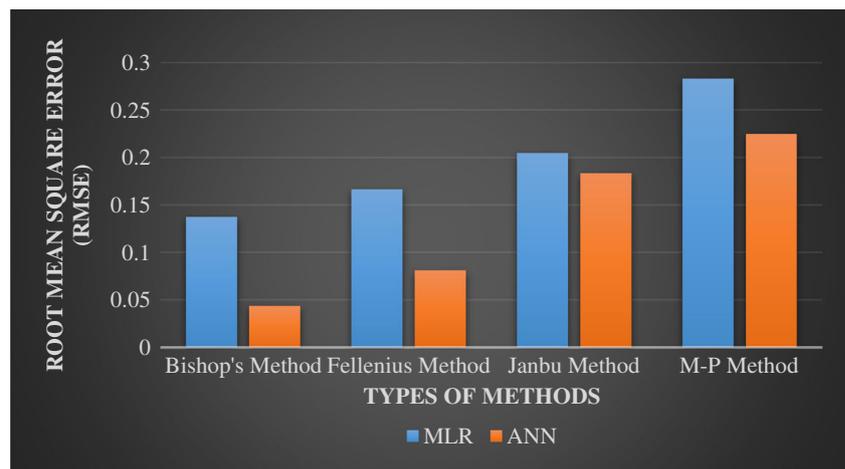


Fig. 8 RMSE vs type of methods for MLR and ANN



ANN is found to be over 99% compared to MLR model having only 96%. Hence, it can be said that the Bishop's model obtained using ANN can give higher correlation compared to the other prediction models.

In the second phase, the stability of the predicted models were checked by performing the error analysis. The error analysis can be performed by computing RMSE and MAE. Based on a logical hypothesis (Smith 1986), if a model gives $R > 0.8$ and the RMSE and MAE values are at the minimum, there is a strong correlation between the predicted values and measured values. It can be observed from Figs. 8 and 9 that RMSE and MAE values were found to be low particularly in case of Bishop's predicted model obtained by using MLR and ANN and are able to predict the target values with acceptable degree of accuracy. On comparing the results obtained by MLR and ANN, it can be further confirmed that the results obtained by ANN are found to be more accurate having lower percentage of errors.

Based on the above analysis, the reasons of frequent landslides occurred around the hills of NH-40 can be explained as follows. The main cause of landslides in these areas is the precipitation. It is found that most of the hill slopes are damp or in a state of wet condition which becomes a problem during the monsoons. Long period of rainfall saturate, soften, and erode the hill slopes leading to instability. Moreover, water enters through the cracks of the soil mass and weakens the underlying soil layer leading to failure of these slopes. It has also come to the notice that the slope cut occurred during the construction of the expressways lead to change in the geometry of the hill slopes causing instability. The results of this study would be very beneficial in the field of decision making and decision support studies for the engineers, planners, developers, etc., by applying the methodology in a geographical information system (GIS) in order to estimate stability for a whole study area and create appropriate landslide hazard assessment maps (Sakellariou and Ferentinou 2001). Some of the pictures of vulnerable sites of NH-40 are shown in Fig. 10.

Fig. 9 MAE vs type of methods for MLR and ANN

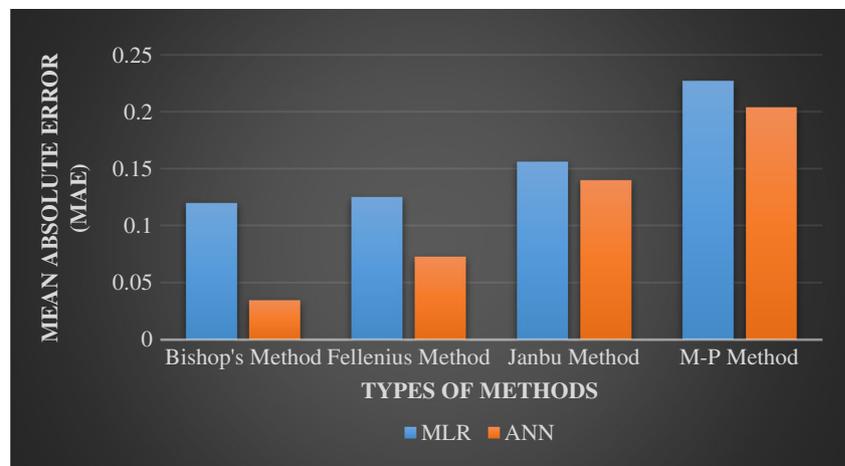


Fig. 10 Vulnerable sites from NH-40



Summary and conclusion

The stability of slopes is a major problem to the geotechnical engineers. Prediction of stability of slopes is a major challenge because the stability of the slopes generally exists as the

combined effects of geology, hydrology, and soil parameters. Predicting the slope stability is an everyday task for the geotechnical engineers. In this paper, 200 artificial slopes were studied and prediction models were developed using MLR and ANN. The validation performance of the prediction

models were done by comparing the predicted results with the analytical results obtained by FEM for 15 vulnerable slopes along NH-40 from Jorabat to Umling. From the presented results, it enables us to draw some interesting conclusions.

- MLR and ANN can act as a good prediction tool for predicting the stability of slopes.
- The FOS obtained by the proposed MLR and ANN models are in general agreement with the results from the FEM analyses. Moreover, Bishop's prediction model is found to be the most accurate compared to other prediction models.
- The parameters of Bishop's prediction model obtained by ANN are found to have a correlation of 99.63% as against 96.14% with MLR.
- Bishop's prediction model obtained by ANN is found to have the lowest values of RMSE and MAE of 0.04 and 0.03, respectively, as against 0.14 and 0.12, respectively with MLR. This illustrates that the proposed models is useful alternatives for slope stability analysis.
- The predicted results of ANN give higher degree of accuracy compared to MLR.
- Finally, the results of this study would be very beneficial in the field of decision making for the engineers, planners, developers, etc., by applying the methodology in a GIS in order to estimate stability for a whole study area and create appropriate landslide hazard assessment maps.

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